Sec. 4.2 Comparing Exponential and Linear Functions

Ex. Identify the linear function and the exponential function from the table below. How can you determine which one is which? Then write a formula for each situation.

Two functions, one linear and one exponential						
x	20	25	30	35	40	45
f(x)	30	45	60	75	90	105
g(x)	1000	1200	1440	1728	2073.6	2488.32

$\frac{\Delta y}{\Delta x} = \frac{45-30}{25-20} = \frac{15}{5}$

$m = 3$

$y = mx + b$

$30 = 3(20) + b$

$30 = 60 + b$

$-30 = b$

$\boxed{f(x) = 3x - 30}$

$\frac{g(25)}{g(20)} = \frac{ab^x}{ab^x} = \frac{\cancel{a}b^{25}}{\cancel{a}b^{20}} = \frac{1200}{1000}$

$b^5 = \frac{6}{5}$

$b = \left(\frac{6}{5}\right)^{\frac{1}{5}} = 1.0371$

$g(20) = ab^{20}$

$\frac{1000}{1.0371^{20}} = \frac{a(1.0371)^{20}}{1.0371^{20}}$

$482.600 = a$

$\boxed{g(x) = 482.600(1.0371)^x}$

NOTE: For a table of data that gives y as a function of x and in which change in x is constant:

a. If the difference of consecutive y values is constant, the table could represent a linear function.

b. If the ratio of consecutive y values is constant, the table could represent an exponential function.

Ex. At time t = 0 years a species of turtle is released into a wetland. When t = 4 years, a biologist estimates there are 300 turtles in the wetland. Three years later, the biologist estimates there are 450 turtles. Let P represent the size of the turtle population in year t.

a. Find a formula for P = f(t) assuming linear growth. Interpret the slope and P intercept of your formula in terms of turtle population.

b. Now find a formula for P = g(t) assuming exponential growth. Interpret the parameters of your formula in terms of the turtle population.

c. In year t = 12, the biologist estimates that there are 900 turtles in the wetland. What does this indicate about the two population models?

a.)

(4, 300) (7, 450)

$m = \frac{450-300}{7-4}$

$= \frac{150}{3}$

$m = 50$

$y = mx + b$

$300 = 50(4) + b$

$300 = 200 + b$

$100 = b$

$\boxed{P = f(t) = 50t + 100}$

m = 50 ⇒ The population of turtles increases by 50 per year

b = 100 ⇒ There were 100 turtles introduced in year 0.

b.) $\frac{f(7)}{f(4)} = \frac{ab^7}{ab^4} = \frac{450}{300}$

$b^3 = 1.5$

$b = (1.5)^{\frac{1}{3}}$

$b = 1.1447$

$f(4) = ab^4$

$\frac{300}{1.1447^4} = \frac{a(1.1447)^4}{1.1447^4}$

$174.72 = a$

$\boxed{P = f(t) = 175(1.1447)^t}$

175 turtles were released in year 0 and they increased by 14.47% per year.

$f(12) = 50(12) + 100$

$= 600 + 100$

$= 700$

$f(12) = 175(1.1447)^{12}$

$= 885.8$

The exponential model was correct

Ex. The population of a colony of rabbits grows exponentially. The colony begins with 10 rabbits; five years later there are 340 rabbits.

(a) Give a formula for the population of the colony of rabbits as a function of the time.

(b) Use a graph to estimate how long it takes for the population of the colony to reach 1000 rabbits.

(0,10) (5,340) $\frac{f(5)}{f(0)} = \frac{ab^5}{ab^0} = \frac{340}{10}$

$b^5 = 34$

$b = 2.0244$

$f(t) = 10(2.0244)^t$

$t = 6.53$ years

($y_2 = 1000$; intersect)

Ex. The following tables contain values for either a linear or exponential model. Determine which would be best and then find a possible formula for the function.

X	F(x)
0	65
1	75
2	85
3	95
4	105`

$m = 10$

$b = 65$

LINEAR

$f(x) = 65 + 10x$

X	G(x)
0	400
1	600
2	900
3	1350
4	2025

$a = 400$

$b = 1.5$

EXPONENTIAL $g(x) = 400(1.5)^x$

Ex. The population of a country is initially 2 million people and is increasing at 4% per year. The country's annual food supply is initially adequate for 4 million people and is increasing at a constant rate adequate for an additional .5 million people per year.

a. Based on those assumptions, in approximately what year would this country first experience shortages of food? 78.32 years later

b. If the country doubles its initial food supply, would shortage still occur? If so, when? 81.40 years

c. If the country doubled the rate at which its food supply increases, in addition to doubling its initial food supply, would shortages still occur? If so, when? 102.23 years

a.) $P(x) = 2(1.04)^x$ $F(x) = 4 + .5x$

Intersect: (78.32, 43.16)

b.) $P(x) = 2(1.04)^x$ $F(x) = 8 + .5x$

Intersect: (81.40, 48.70)

c.) $P(x) = 2(1.04)^x$ $F(x) = 8 + x$

Intersect: (102.23, 110.23)

HW: pg 145 – 148, #3-45 (m/3)